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ORIENTAL JOURNAL OF CHEMISTRY
An International Open Free Access, Peer Reviewed Research Journal

www.orientjchem.org

ISSN: 0970-020 X
CODEN: OJCHEG
2017, Vol. 33, No. (2):
Pg. 951-962

Study of Correlation Between the Atomic Numbers and the Atomic Weights of Elements in the Periodic Table With Sierpinski Triangle Fractals

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http://dx.doi.org/10.13005/ojc/330247

(Received: February 02, 2017; Accepted: April 09, 2017)

ABSTRACT

Waclaw Sierpinski described fractal geometries such as Sierpinski triangle, gasket and carpet. In this paper, Sierpinski triangle is used to find any equation between the atomic number and the atomic weight of elements in the periodic table. First by using Matlab program, an algorithm is written to create a right angle triangle between the atomic numbers and atomic weights. Then this original triangle is divide to 8 smaller triangles on the hypotenuse of the original triangle to get more accurate results and reduce errors. Finally, equations of correlation between the atomic numbers and the atomic weights of elements are obtained to calculate the atomic weights of the elements in eighth period .

Keywords: Sierpinski's fractals, Self-similarity, The periodic table of the elements, Atomic number, Atomic weight, Right angled triangle, Matlab program.

INTRODUCTION

In 1975 Benoit Mandelbrot introduced a new branch of geometry known as fractal geometry to find order in chaotic shapes. In fact, fractal geometries describe the fractals which have complex geometric structures in natural and physical sciences. Most of fractal objects are self-similar in nature which means that if a tiny portion of a geometric structure is enlarged, an analogous structure of the whole is obtained. This means that the fractal objects could be broken into even finer pieces having same features as the original and with dimension always less than its original. The dimension of a fractal object is usually not an integer but fractional. Fractal objects are omni present in nature and can be well-approximated mathematically. There are some mathematically developed fractals such as Cantor set by George Cantor, Peano's curve by Giuseppe Peano, Koch's curve by Helge von Koch in, Sierpinski's fractals such as carpet, triangle, etc., by Waclaw Sierpinski in, Julia set by Gaston Julia [1].

The triangle known as Sierpinski triangle can be considered as the composition of three small equal triangles, each of them is exactly half the size and of the self-similar copies original triangle. Thus if we magnify any of these three triangles by a factor of 2, we will get the original triangle. Again, each of these three small self-similar triangles can be considered as the combination of another three small self-similar triangles. This type of self-similarity is known as exact self-similarity. It is the strongest self-similarity occurred in fractal images [1]. The Sierpinski triangle is a canonical starting point for many researches for fractals and self-assembly. Winfree showed that the Sierpinski triangle weakly self-assembles [2].

Sierpinski gasket and carpet were discovered by the Polish mathematician Waclaw

Sierpinski in 1916 .Imagine filled equilateral triangle such as S_0 with each side of unit length. Now divide this triangle into four equal small equilateral triangles using the midpoints of the three vertices of the original triangle S_0 as new vertices and remove the interior of the middle triangle. The result is a triangle called S_1. Repeat this process in each of the remaining three equal triangles to produce the triangle S_2. Repeat this process continuously and finally the Sierpinski triangle is formed. The figure1 displays five steps of Sierpinski triangle. [1].

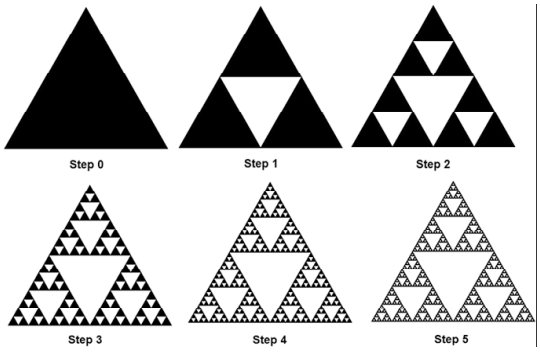

Fig. 1: Construction of Sierpinski triangle [1]

We start by labeling p_1, p_2 and p_3 as the corners of the Sierpinski triangle, and a random point v_1. Set $v_n+1 = ½ (v_n + p_{r\,n})$, where rn is a random number 1, 2 or 3. If the first point v_1 was a point on the Sierpiñski triangle, then all the points vn lie on the Sierpinski triangle. If the first point v_1 to lie within the perimeter of the triangle is not a point on the Sierpinski triangle, none of the points vn will lie on the Sierpinski triangle, however they will converge on the triangle. If v_1 is outside the triangle, the only way vn will land on the triangle, is the triangle was infinitely large. The algorithm can be applied to any kind of triangle. Also, the same rule can be applied to other objects like pyramids, cubes, etc. The above algorithm is not the only method to draw a Sierpinski triangle. There is a method called iterated function systems (IFS) [3]. The number of triangles in a Sierpinski triangle can be found by using the formula $N_n=3^n$, where N is the number of triangles and n the number of iterations [4].

In this research we uase righ-angled Sierpinski triangled which can be called as disceret Sierpinski triangle. To draw this triangle, first we drew a triangle with vertices (0, 0), (1, 0), and (0, 1). Then it will be devided to a smaller triangle with vertices (3⁄4, 3⁄4), (1, 3⁄4), and (3⁄4, 1).Then three smaller triangles, one with vertices (3⁄8, 3⁄8),(1⁄2, 3⁄8), (3⁄8, 1⁄2), one with (7⁄8, 3⁄8), (1, 3⁄8), (7⁄8, 1⁄2), and one with vertices (3⁄8, 7⁄8), (1⁄2,7⁄8), (3⁄8, 1) [4]. The whole shape can be seen as it is made of three pieces scaled by 1⁄2 and one piece scaled by 1⁄4. The result suggests defining a process: replace a shape by three copies scaled by 1⁄2 and one copy scaled by 1⁄4. Taking the base and altitude of the fractal to be 1, the process consists of four transformations as functions below and figure 2[5].

$T1(x, y) = (x⁄2, y⁄2)$
$T2(x, y) = (x⁄2, y⁄2) + (1⁄2, 0)$
$T3(x, y) = (x⁄2, y⁄2) + (0, 1⁄2)$
$T4(x, y) = (x⁄4, y⁄4) + (3⁄4, 3⁄4)$. [5]

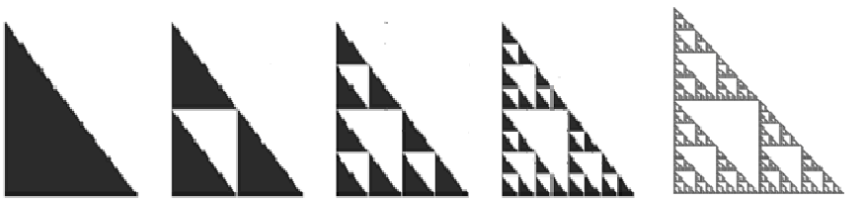

Fig. 2: Construction of righ-angled Sierpinski triangle [5],[6]

The periodic table of elements was create by Russian scientist Dmitri Ivanovich Mendeleev . In this research, we try to perform Sierpinski triangled algorithm on this table in which the chemical elements are arranged by order of atomic number in such a way that the periodic properties (chemical periodicity) of the elements are made clear. In this research, we used righ-angled Sierpinski triangled in order to find the corrolation between atomic number and atomic weight of 118 elements in the periodic table.

Computional method

This method provides the graphical analysis of the behaviour between atomic number and atomic weight , so MATLAB 2013 software is used for writing the right angle triangle algorithm and analyzing graphically the corrolation between atomic number and atomic weight. In th first step, by using the software , we drew a triangle with vertices as below:

```
px(1)=0;
px(2)=120;
px(3)=120;
py(1)=0;
py(2)=0;
py(3)=310;
vx_in=60;
vy_in=145;
```

In the next step, we divide the triangle into smaller triangles as seven period of the periodic table to get more accuracy in results. Each triangle are different in vertices , vx and Vy. In each step we study the hypotenuse of the triangles to determine the relationship between atomic number and atomic weight of 118 elements and more accurate results.

RESULTS and DISCUSSIONS

The initial triangle

The main purpose in this study is determination of the atomic weight of elements by their atomic nomber with a righ-angled Sierpinski triangle algorithm. To achieve this goal, we used Matlab 2013 software to write the triangle algorithm with vertices, vx and vy as blow:

```
px(1)=0;
px(2)=120;
px(3)=120;
py(1)=0;
py(2)=0;
py(3)=310;
vx_in=60;
vy_in=155;
```

By running the progrom, the results are shown as figure(3) which the atomic number of elements are as axis (X).The origin right angled triangle is divided to other triangles and showed by white color triangles which in these triangles, there are no points. On the contrary, the blue parts are consisted of points. Each point shows an atomic number and atomic weight. The weights of elements which are resulted by Sierpinski algorithm are near and along the hypotenuse of the original triangle. Some of the atomic weights are resulted in white triangles and because there are no points in them, they cannot be accepted and are shown with red points . So the program has chosen the nearest point for those atomic weight in blue parts. The atomic weights which are chosen by the program can be seen with green points in figure (3).

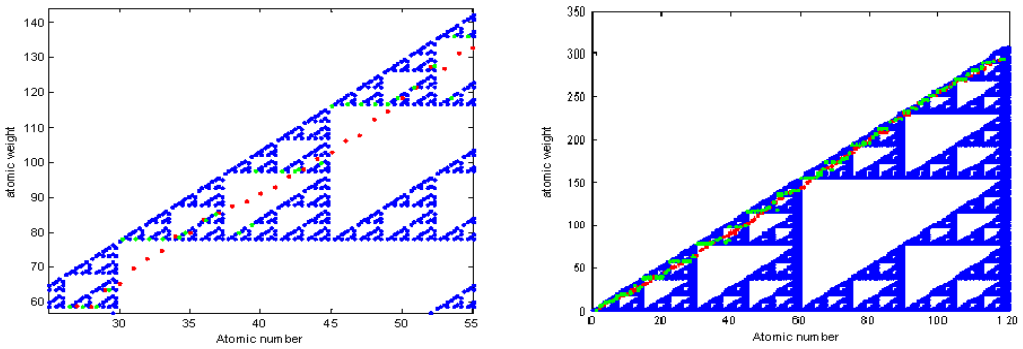

Fig. 3: The initial triangle

Table1: Atomic Symbol, Atomic Number, Atomic Weight , Calculated Atomic Number and Calculated Atomic Weight by initial Sierpinski right angled triangle algorithm

Atomic Symbol	Atomic Number	Atomic Weight Calculated	Atomic Number Calculated	Atomic Weight	Atomic Symbol	Atomic Number	Atomic Weight Calculated	Atomic Number	Calculated Atomic Weight
H	1	1.00794	1.0000	1.0000	Rh	45	102.90550	45.0000	116.2500
He	2	4.002602	2.1094	5.44922	Pd	46	106.421	46.1719	116.8555
Li	3	6.941	2.5781	6.66016	Ag	47	107.8682	47.1094	116.8555
Be	4	9.012183	3.7500	9.68750	Cd	48	112.411	48.0469	116.8555
B	5	10.811	5.1562	10.8984	In	49	114.818	48.9844	116.8555
C	6	12.0107	6.3281	11.5039	Sn	50	118.710	49.6875	118.6719
N	7	14.0067	7.0312	13.3203	Sb	51	121.760	50.8594	121.6992
O	8	15.9994	7.5000	19.3750	Te	52	127.603	52.2656	127.7539
F	9	18.9984032	8.6719	19.9805	I	53	126.90447	53.4375	118.6719
Ne	10	20.1797	9.6094	19.9805	Xe	54	131.293	53.6719	136.2305
Na	11	22.9897693	11.0156	22.4023	Cs	55	132.905452	54.6094	136.2305
Mg	12	24.3050	12.4219	29.6680	Ba	56	137.327	56.0156	137.4414
Al	13	26.9815386	13.3594	24.8242	La	57	138.90547	57.4219	138.6523
Si	14	28.0855	14.2969	27.2461	Ce	58	140.116	58.1250	140.4688
P	15	30.973762	15.0000	38.7500	Pr	59	140.90765	59.2969	141.0742
S	16	32.065	16.1719	39.3555	Nd	60	144.242	60.2344	155.6055
Cl	17	35.453	17.1094	39.3555	Pm	61	145	61.1719	155.6055
Ar	18	39.948	18.2812	39.9609	Sm	62	150.362	62.1094	155.6055
K	19	39.0983	18.9844	39.3555	Eu	63	151.964	63.0469	155.6055
Ca	20	40.078	20.1562	39.9609	Gd	64	157.253	64.4531	156.8164
Sc	21	44.955912	21.0937	44.8047	Tb	65	158.92535	65.1562	158.6328
Ti	22	47.867	22.0312	47.2266	Dy	66	162.500	66.3281	161.6602
V	23	50.9415	22.5000	58.1250	Ho	67	164.93032	66.7969	165.2930
Cr	24	51.9961	23.6719	58.7305	Er	68	167.259	67.5000	174.3750
Mn	25	54.938045	24.6094	58.7305	Tm	69	168.93421	69.0000	164.6875
Fe	26	55.845	26.4844	58.7305	Yb	70	173.054	69.6094	174.9805
Co	27	58.933195	27.4219	58.7305	Lu	71	174.9668	71.4844	174.9805
Ni	28	58.6934	28.3594	58.7305	Hf	72	178.492	72.4219	177.4023
Cu	29	63.546	29.2969	63.5742	Ta	73	180.94788	73.3594	179.8242
Zn	30	65.382	30.2344	78.1055	W	74	183.841	74.2969	184.6680
Ga	31	69.723	31.1719	78.1055	Re	75	186.207	75.0000	193.7500
Ge	32	72.641	32.1094	78.1055	Os	76	190.233	76.1719	194.3555
As	33	74.92160	33.0469	78.1055	Ir	77	192.217	77.1094	194.3555
Se	34	78.963	34.2187	78.7109	Pt	78	195.084	78.2812	194.9609
Br	35	79.904	34.6875	79.9219	Au	79	196.966569	78.5156	196.7773
Kr	36	83.798	36.0937	83.5547	Hg	80	200.592	80.1562	197.3828
Rb	37	85.4678	36.7969	85.3711	Tl	81	204.3833	81.0937	204.6484
Sr	38	87.621	38.4375	79.9219	Pb	82	207.210	82.0312	207.0703
Y	39	88.90585	39.3750	82.3437	Bi	83	208.98040	82.5000	213.1250
Zr	40	91.224	39.6094	97.4805	Po	84	209	83.6719	213.7305
Nb	41	92.90638	41.4844	97.4805	At	85	210	84.6094	213.7305
Mo	42	95.962	42.4219	97.4805	Rn	86	222	85.7812	221.6016
Tc	43	98	42.6562	98.0859	Fr	87	223	87.4219	223.4180

Ru Atomic Symbol	44 Atomic Number	101.072 Atomic Weight	44.2969 Calculated Atomic Number	99.9023 Calculated Atomic Weight	Ra Atomic Symbol	88 Atomic Number	226 Atomic Weight	87.8906 Calculated Atomic Number	225.8398 Calculated Atomic Weight
Ac	89	227	89.2969	225.8398	Rf	104	267	104.2969	267.0117
Th	90	232.03806	90.0000	232.5000	Db	105	268	105.0000	271.2500
Pa	91	231.03588	91.1719	233.1055	Sg	106	271	106.1719	271.8555
U	92	238.02891	92.1094	237.9492	Bh	107	272	107.1094	271.8555
Np	93	237	93.2812	236.1328	Hs	108	270	108.0469	271.8555
Pu	94	244	94.4531	244.0039	Mt	109	276	108.5156	276.6992
Am	95	243	94.9219	242.7930	Ds	110	281	109.9219	281.5430
Cm	96	247	95.6250	247.0313	Rg	111	280	110.8594	281.5430
Bk	97	247	96.7969	247.6367	Cn	112	285	112.0313	284.5703
Cf	98	251	97.5000	251.8750	Nh	113	284	112.5000	290.6250
Es	99	252	98.6719	252.4805	Fl	114	289	113.6719	291.2305
Fm	100	257	99.6094	257.3242	Mc	115	288	114.6094	291.2305
Md	101	258	101.0156	258.5352	Lv	116	293	116.0156	292.4414
No	102	259	102.0000	261.5625	Ts	117	294	117.2000	293.0000
Lr	103	262	103.3594	262.1680	Og	118	294	118.1250	295.4688

The results of the Sierpinski algorithm are shown in table (1) which atomic symbols, atomic Numbers, atomic weights of the periodic table and also calculated atomic numbers and atomic weights by Sierpinski right angled triangle algorithm are demonstrated. As it is shown in the table (1) the calculated atomic numbers and atomic weights are not exactly same as atomic numbers and atomic weights in the periodic table and there are more or less errors in them also table (1) shows that for some of elements, calculated atomic weights are the same beacause the atomic weights of those elements have resulted in the white triangles which there are no points. So the program had to choose the same number for their weights in blue parts of the the initial triangle, where there are points and numbers.

Calculation in the blue parts is the main reason of occuring error in atomic Numbers and atomic weights which are resulted by Sierpinski right angled triangle algorithm. The percentage of absolute errors in obtained atomic Numbers and atomic weights are demonstrated in table (2). In table (2), Helium, Lithium, Beryllium, Boron, Carbon, Fluorine, Neon, Magnesium ,Aluminum and silicon have the highest percentage of absolute errors in obtained atomic number while Helium, Beryllium, Oxygen, Magnesium, Aluminum, Phosphorus, Sulfur, chlorine, Vanadium, Chromium, Manganese, Iron, Zinc, Gallium, Germanium, Strontium, Yttrium, Zirconium, Rhodium, Palladium , Silver, Neodymium and Promethium have the highest percentage of absolute errors in obtained atomic weights.

The interesting point is that smaller and lighter elements have the highest errors but heavier elements have shown much less errors in results. In this method, the average of % error for the calculated Atomic Numbers is % 0.898 and the average of %error for the calculated Atomic weights is % 3.4568.

The Seven period triangles

There are seven period in the periodic table of elements. In this step, for each period of the periodic table, we create a triangle exept for the seventh period which we make two triangles. For this purpose, the Sierpinski right angled triangle algorithm is slightly changed, so the hypotenuse of the original triangle are divide to 8 right angled triangle. Table 3 shows the changes with vertices, vx and Vy to make 8 triangles out of the origin one. This process is done to see the effect of dividing the original on accuracy of calculated atomic number and atomic weight of elements and % errors for both these factors.

Table 2: % Error in calculated Atomic Number and calculated Atomic Weight by initial Sierpinski right angled triangle algorithm

Atomic Symbol	% Error Atomic in Number	% Error in Atomic WeighT	Atomic Symbol	% Error in Atomic Number	% Error in Atomic Weight	Atomic Symbol	% Error In Atomic Number	% Error in Atomic Weight
H	0.0000	0.7900	Nb	1.1814	4.9233	Tl	0.1157	0.1297
He	5.4700	36.1419	Mo	1.0045	1.5824	Pb	0.0381	0.0674
Li	14.0625	4.0462	Tc	0.7994	0.0876	Bi	0.6024	1.9832
Be	6.2500	7.4934	Ru	0.7647	1.1572	Po	0.3906	2.2634
B	3.1250	0.8088	Rh	0.0000	12.9677	At	0.4595	1.7764
C	5.4687	4.2195	Pd	0.3736	9.8049	Rn	0.2544	0.1794
N	0.4464	4.9004	Ag	0.2327	8.3317	Fr	0.4849	0.1874
O	6.2500	21.0982	Cd	0.0976	3.9537	Ra	0.1243	0.0709
F	3.6458	5.1692	In	0.0319	1.7745	Ac	0.3336	0.5111
Ne	3.9062	0.9872	Sn	0.6250	0.0321	Th	0.0000	0.1991
Na	0.1421	2.5530	Sb	0.2757	0.0499	Pa	0.1888	0.8958
Mg	3.5156	22.0653	Te	0.5108	0.1182	U	0.1189	0.0335
Al	2.7644	7.9955	I	0.8255	6.4872	Np	0.3024	0.3659
Si	2.1205	2.9887	Xe	0.6076	3.6707	Pu	0.4820	0.0016
P	0.0000	25.1058	Cs	0.7102	2.5018	Am	0.0822	0.0852
S	1.0742	22.7365	Ba	0.0279	0.0833	Cm	0.3906	0.0127
Cl	0.6434	11.0074	La	0.7401	0.1822	Bk	0.2094	0.2577
Ar	1.5625	‘0.0323	Ce	0.2155	0.2518	Cf	0.5102	0.3486
K	0.0822	0.6577	Pr	0.5032	0.1182	Es	0.3314	0.1907
Ca	0.7812	0.2921	Nd	0.3906	7.8781	Fm	0.3906	0.1261
Sc	0.4464	0.3364	Pm	0.2818	7.3141	Md	0.0154	0.2074
Ti	0.1420	1.3379	Sm	0.1764	3.4872	No	0.0000	0.9894
V	2.1739	14.1015	Eu	0.0744	2.3963	Lr	0.3489	0.0641
Cr	1.3671	12.9517	Gd	0.7080	0.2776	Rf	0.2855	0.0044
Mn	1.5625	6.9031	Tb	0.2403	0.1841	Db	0.0000	1.2127
Fe	1.8630	5.1669	Dy	0.4972	0.5168	Sg	0.1621	0.3157
Co	1.5625	0.3439	Ho	0.3032	0.2199	Bh	0.1025	0.0531
Ni	1.2835	0.0631	Er	0.7353	4.2544	Hs	0.0434	0.6872
Cu	1.0237	0.0444	Tm	0.0000	2.5138	Mt	0.4444	0.2533
Zn	0.7813	19.4602	Yb	0.5580	1.1132	Ds	0.0710	0.1932
Ga	0.5544	12.0225	Lu	0.6822	0.0078	Rg	0.1266	0.5511
Ge	0.3418	7.5225	Hf	0.5859	0.6105	Cn	0.0279	0.1508
As	0.1421	4.2496	Ta	0.4923	0.6209	Nh	0.4424	2.3327
Se	0.6434	0.3192	W	0.4012	0.4498	Fl	0.2878	0.7718
Br	0.8928	0.0224	Re	0.0000	4.0509	Mc	0.3396	1.1217
Kr	0.2604	0.2903	Os	0.2261	2.1671	Lv	0.0134	0.1906
Rb	0.5489	0.1131	Ir	0.1420	1.1125	Ts	0.1709	0.3401
Sr	1.1513	8.7868	Pt	0.3606	0.0631	Og	0.1059	0.4996
Y	0.9615	7.3809	Au	0.6131	0.0961			
Zr	0.9765	6.8583	Hg	0.1953	1.5999			

Table 3: Vertices , vx and vy of triangles for the seven period

First Period	Second Period	Third Period	Fourth Period
px(1)=0;	px(1)=3;	px(1)=11;	px(1)=19;
px(2)=3;	px(2)=11;	px(2)=19;	px(2)=37;
px(3)=3;	px(3)=11;	px(3)=19;	px(3)=37;
py(1)=0;	py(1)=6.5;	py(1)=23;	py(1)=39;
py(2)=0;	py(2)=6.5;	py(2)=23;	py(2)=39;
py(3)=6;	py(3)=23;	py(3)=39.95;	py(3)=85.47;
vx_in=1.6	vx_in=7;	vx_in=16;	vx_in=28;
vy_in=2.9	vy_in=13;	vy_in=32;	vy_in=62
Fifth Period	**Sixth Period**	**Seventh Period(a)**	**Seventh Period(b)**
px(1)=37;	px(1)=55;	px(1)=87;	px(1)=104;
px(2)=55;	px(2)=87;	px(2)=105;	px(2)=118;
px(3)=55;	px(3)=87;	px(3)=105;	px(3)=118;
py(1)=85.47;	py(1)=132.9;	py(1)=223;	py(1)=267;
py(2)=85.47;	py(2)=132.9;	py(2)=223;	py(2)=267;
py(3)=132.9;	py(3)=223;	py(3)=267;	py(3)=294;
vx_in=48;	vx_in=79;	vx_in=100;	vx_in=114;
vy_in=112.4	vy_in=197;	vy_in=257;	vy_in=289;

Table 4: Atomic Symbol, Atomic Number, Atomic Weight , calculated Atomic Number and calculated Atomic Weight by eight Sierpinski right angled triangles algorithm

Atomic Symbol	Atomic Number	Atomic Weight	Calculated Atomic Number	Calculated Atomic Weight	Atomic Symbol	Atomic Number	Atomic Weight	Calculated Atomic Number	Calculated Atomic Weight
H	1	1.00794	1.0000	1.0000	Rh	45	102.90550	44.9180	103.3614
He	2	4.002602	2.0047	4.0070	Pd	46	106.421	46.0430	109.2902
Li	3	6.941	3.2187	6.9375	Ag	47	107.8682	47.1680	109.2902
Be	4	9.012183	4.2187	9.0000	Cd	48	112.411	48.0820	112.4398
B	5	10.811	5.0937	10.8047	In	49	114.818	49.4180	115.2189
C	6	12.0107	5.6875	12.0156	Sn	50	118.710	50.1641	118.6591
N	7	14.0067	7.0156	14.7754	Sb	51	121.760	50.8437	121.8841
O	8	15.9994	7.6250	15.9844	Te	52	127.603	52.4141	126.0700
F	9	18.9984032	9.0625	18.9766	I	53	126.90447	52.6953	126.8111
Ne	10	20.1797	9.6406	20.1895	Xe	54	131.293	54.3828	131.2577
Na	11	22.9897693	11.0195	23.0352	Cs	55	132.905452	55.0937	133.1504
Mg	12	24.3050	11.6562	24.3406	Ba	56	137.327	56.4375	136.9203
Al	13	26.9815386	13.0195	27.2727	La	57	138.90547	57.0937	138.7816
Si	14	28.0855	14.4141	28.1023	Ce	58	140.116	57.5937	140.1895
P	15	30.973762	15.0195	31.5102	Pr	59	140.90765	59.0937	144.4129
S	16	32.065	16.2891	32.0750	Nd	60	144.242	60.0937	144.4129
Cl	17	35.453	17.0195	35.7477	Pm	61	145	61.3437	145.1168
Ar	18	39.948	18.4766	38.8285	Sm	62	150.362	62.2187	150.3961
K	19	39.0983	19.0352	39.0898	Eu	63	151.964	63.0937	155.6754
Ca	20	40.078	19.9844	40.0855	Gd	64	157.253	63.6875	157.3336

Sc	21	44.955912	21.3203	44.9884	Tb	65	158.92535	64.7187	158.8430
Ti	22	47.867	22.4453	47.8928	Dy	66	162.500	65.5937	162.7145
V	23	50.9415	23.4297	50.4341	Ho	67	164.93032	67.0937	166.9379
Cr	24	51.9961	24.0625	52.0550	Er	68	167.259	68.2187	167.2898
Mn	25	54.938045	25.1875	54.9594	Tm	69	168.93421	68.8437	169.0496
Fe	26	55.845	25.5391	55.8798	Yb	70	173.054	70.2187	172.9211
Co	27	58.933195	26.7344	58.9639	Lu	71	174.9668	70.9375	174.9313
Ni	28	58.6934	28.0352	62.3248	Hf	72	178.492	72.1875	178.4508
Cu	29	63.546	29.0547	63.5038	Ta	73	180.94788	72.5937	181.0160
Zn	30	65.382	30.0742	65.4108	W	74	183.841	74.0937	183.8316
Ga	31	69.723	30.8828	69.6756	Re	75	186.207	74.9375	186.1938
Ge	32	72.641	32.0430	72.6717	Os	76	190.233	76.3750	190.2141
As	33	74.92160	32.9219	74.9380	Ir	77	192.217	76.5937	192.2785
Se	34	78.963	34.4687	78.9278	Pt	78	195.084	78.0937	195.0941
Br	35	79.904	34.8555	79.9326	Au	79	196.966569	79.0937	200.7254
Kr	36	83.798	36.3672	83.8344	Hg	80	200.592	80.0937	200.7254
Rb	37	85.4678	37.0430	85.5752	Tl	81	204.3833	80.8750	204.2922
Sr	38	87.621	37.7891	87.5331	Pb	82	207.210	82.3750	207.1078
Y	39	88.90585	38.8711	88.9101	Bi	83	208.98040	83.0000	210.0000
Zr	40	91.224	40.4180	91.5039	Po	84	209	84.0937	211.9879
Nb	41	92.90638	40.9805	92.9861	At	85	210	85.0937	211.9879
Mo	42	95.962	41.5430	97.4327	Rn	86	222	86.4375	221.3891
Tc	43	98	42.8516	97.9084	Fr	87	223	87.0508	223.1328
Ru	44	101.072	43.5312	101.1334	Ra	88	226	88.2266	226.0156

Atomic Symbol	Atomic Number	Atomic Weight	Calculated Atomic Number	Calculated Atomic Weight	Atomic Symbol	Atomic Number	Atomic Weight	Calculated Atomic Number	Calculated Atomic Weight
Ac	89	227	89.1719	226.9688	Rf	104	267	104.0391	267.0859
Th	90	232.03806	90.4961	231.5547	Db	105	268	105.3906	268.0156
Pa	91	231.03588	91.4102	231.0391	Sg	106	271	106.0625	270.9883
U	92	238.02891	92.4687	236.4375	Bh	107	272	107.4375	271.9844
Np	93	237	92.7266	237.0156	Hs	108	270	108.4688	268.8789
Pu	94	244	94.4336	241.1797	Mt	109	276	109.1094	276.0313
Am	95	243	95.2070	243.0703	Ds	110	281	110.4922	279.5313
Cm	96	247	96.4844	246.2188	Rg	111	280	111.0391	280.5859
Bk	97	247	96.8047	246.9844	Cn	112	285	112.4688	283.3750
Cf	98	251	98.4414	250.9766	Nh	113	284	112.7891	283.9609
Es	99	252	98.8633	252.0078	Fl	114	289	114.4688	287.2109
Fm	100	257	100.4609	255.9219	Mc	115	288	114.8750	288.0156
Md	101	258	101.3047	257.9844	Lv	116	293	116.0000	291.5000
No	102	259	101.7266	259.0156	Ts	117	294	117.0000	292.8000
Lr	103	262	102.5000	262.0000	Og	118	294	118.0000	293.9000

The Calculated atomic numbers and atomic weights resulted by eight right-angle Sierpinski triangles algorithm are shown in table 4. Comparing this table with table 1, shows that the numbers specially calculated atomic weights are more acurrate and closer to real ones. Also, there are less elements with same atomic weights in this method the groups with such condition have shifted to heavier elements. Figure 4 shows the triangles for the seven period, which there are six triangles for the

Table 5: % Error in calculated Atomic Number and calculated Atomic Weight by seven period Sierpinski right angled triangles algorithm

Atomic Symbol	% Error in Atomic Number	% Error in Atomic Weight	Atomic Symbol	% Error in Atomic Number	% Error In Atomic Weight	Atomic Symbol	% Error in Atomic Number	% Error in Atomic Weight
H	0.0000	0.7877	Nb	0.0476	0.0858	Tl	0.1543	0.0446
He	0.2350	0.1099	Mo	1.0881	1.5326	Pb	0.4573	0.0493
Li	7.2900	0.0504	Tc	0.3451	0.0935	Bi	0.0000	0.4879
Be	5.4675	0.1352	Ru	1.0654	0.0607	Po	0.1115	1.4296
B	1.8740	0.0583	Rh	0.1822	0.4430	At	0.1102	0.9466
C	5.2083	0.0408	Pd	0.0935	2.6961	Rn	0.5087	0.2752
N	0.2257	5.4881	Ag	0.3574	1.3183	Fr	0.0584	0.0595
O	4.6875	0.0937	Cd	0.1708	0.0256	Ra	0.2575	0.0069
F	0.6944	0.1148	In	0.8531	0.3492	Ac	0.1932	0.0137
Ne	3.5940	0.0486	Sn	0.3282	0.0429	Th	0.5512	0.2083
Na	0.1773	0.1976	Sb	0.3065	0.1019	Pa	0.4508	0.0014
Mg	2.8650	0.1465	Te	0.7963	1.2014	U	0.5094	0.6686
Al	0.1500	1.0791	I	0.5749	0.0736	Np	0.2940	0.0066
Si	2.9578	0.0598	Xe	0.7089	0.0269	Pu	0.4613	1.1559
P	0.1300	1.7319	Cs	0.1704	0.1843	Am	0.2179	0.0289
S	1.8069	0.0312	Ba	0.7812	0.2961	Cm	0.5046	0.3163
Cl	0.1147	0.8312	La	0.1644	0.0892	Bk	0.2013	0.0063
Ar	2.6478	2.8024	Ce	0.7005	0.0524	Cf	0.4504	0.0093
K	0.1853	0.0217	Pr	0.1589	2.4876	Es	0.1381	0.0031
Ca	0.0780	0.0187	Nd	0.1562	0.1185	Fm	0.4609	0.4195
Sc	1.5252	0.0723	Pm	0.5634	0.0805	Md	0.3017	0.0061
Ti	2.0241	0.0539	Sm	0.3527	0.0227	No	0.2680	0.0060
V	1.8682	0.9960	Eu	0.1487	2.4423	Lr	0.4854	0.0000
Cr	0.2604	0.1133	Gd	0.4883	0.0512	Rf	0.0376	0.0322
Mn	0.7500	0.0389	Tb	0.4328	0.0518	Db	0.3720	0.0058
Fe	1.7727	0.0623	Dy	0.6156	0.3120	Sg	0.0590	0.0043
Co	0.9837	0.0521	Ho	0.1398	1.2172	Bh	0.4089	0.0057
Ni	0.1257	6.1871	Er	0.3216	0.0184	Hs	0.4341	0.4152
Cu	0.1886	0.6641	Tm	0.2265	0.0683	Mt	0.1004	0.0113
Zn	0.2473	0.0440	Yb	0.3124	0.0768	Ds	0.4474	0.5227
Ga	0.3781	0.0680	Lu	0.0880	0.0203	Rg	0.0352	0.2092
Ge	0.1344	0.0423	Hf	0.2604	0.0231	Cn	0.4186	0.5702
As	0.2367	0.0219	Ta	0.5566	0.0371	Nh	0.1866	0.0138
Se	1.3785	0.0446	W	0.1266	0.0051	Fl	0.4112	0.6191
Br	0.4128	0.0358	Re	0.0833	0.0071	Mc	0.1087	0.0054
Kr	1.0200	0.0434	Os	0.4934	0.0099	Lv	0.0000	0.5119
Rb	0.1162	0.1257	Ir	0.5277	0.0320	Ts	0.0000	0.4082
Sr	0.5550	0.1003	Pt	0.1201	0.0052	Og	0.0000	0.0340
Y	0.3305	0.0048	Au	0.1186	1.9084			
Zr	1.0450	0.3068	Hg	0.1171	0.0665			

periods 1-6 and two triangles for the period 7 which it is done to see better results in the last period. The atomic weights of 118 elements are shown as red points. As it is seen, the hypotenuse of the connected triangles is not a stright line, that it is a curve. Also with comparing tables 2 and 5, we see that %error for some elements such as Helium, Lithium, Oxygen, Fluorine, Magnesium, Aluminium, Silicon, Chlorine, Vanadium, Argon , Chromium, manganese, Iron, Zinc, Galium, Germanium, Rhodium, Paladium and Silver % errors in Atomic weight and atomic number are reduced. For Some elemnts such as

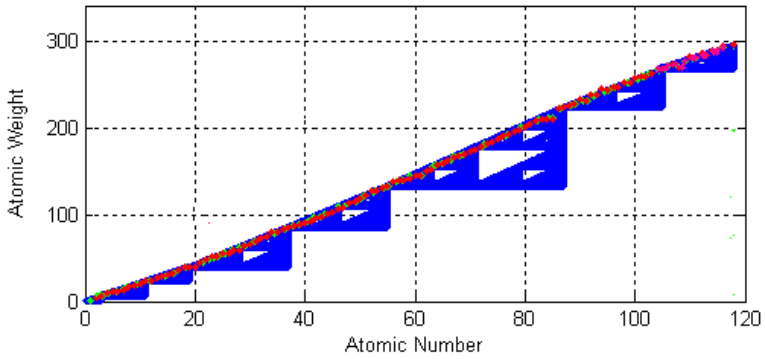

Fig. 4: The eight triangles fot the seven periods of the periodic table

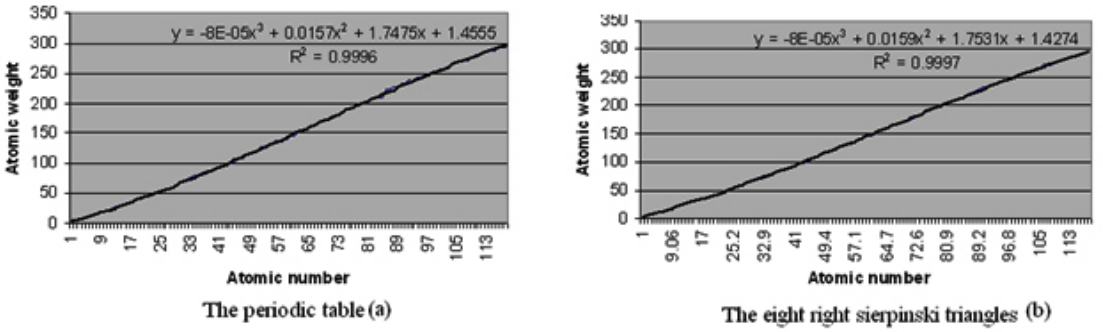

Fig.5: plot and equation for atomic numbers and atomic weights for (a) the periodic table of elements (b) the eight right angles Sierpinski triangles

Table 6: The atomic numbers of eight period elements and their calculated atomic weights with equation 3

Atomic Number	Atomic Weight	Atomic Number	Atomic Weight	Atomic Number	Atomic Weight	Atomic Number	Atomic Weight
117	283.746	126	300.428	135	314.641	144	326.035
118	285.711	127	302.136	136	316.053	145	327.112
119	287.649	128	303.812	137	317.429	146	328.149
120	289.559	129	305.457	138	318.769	147	329.146
121	291.443	130	307.07	139	320.074	148	330.103
122	293.298	131	308.651	140	321.341	149	331.018
123	295.124	132	310.199	141	322.572	150	331.892
124	296.922	133	311.714	142	323.765		
125	298.69	134	313.195	143	324.919		

Nitrogen, Nickel, Selenium, Barium and Gold, there are increased % errors in their atomic numbers or atomic weights. However, In this method, the average of % errors for the calculated atomic Numbers and the calculated atomic weights, respectively, are reduced to % 0.6867 and % 0.4113. This means that dividing the hypotenuse of the initial triangle to eight triangles was a successful method to reduce % errors in resulted numbers.

Comparing the equations

In this level,by using Excel- 2016 program and the numbers in tables 1 and 4, we try to drew two plots and achieve their equations. In the first plot, the atomic numbers and the atomic weights in the periodic table of elements are used , respectively, as axis X and Y. The best resulted equation between these two parameter is a third degree equation (eq.1). The curve is shown in figure(5).

$$y = -8*10^{-5}x3+0.0157x2+1.7475x+1.4555 \quad ...(1)$$

In the second plot, calculated atomic numbers and atomic weights reslted of eight right angled Sierpinski triangles method are used, respectively, as axis X and Y. The best resulted equation between these two parameter is also third degree equation (eq.2) which its curve is showed in figure 5.

$$y = -8* 10\text{-}5x3+0.0159x2+1.7531x+1.4274 \quad ...(2)$$

Comparing the two equations shows there are slight difference between them . Also, the R- squared value for equations 1 and 2 are 0.996 and 0.997 which means, there is almost perfect correllation between the atomic numbers and the atomic weights in the two equations. Unfortunately, Hydrogen dose not answer right in these two equations, we can consider that it is because Hydrogen dose not have neutron but other elements have. So, we cosider Hydrogen out of any equation, as it is always considered seperate of any group in the periodic table of elements. As for elements with atomic number 117 and 118 , a slight change in the slop of curve can be seen in figure 5a and 5b. If we consider equation 1 and place atomic numbers 117 and 188, the calculated atomic weights are, respectively , 296.0662 and 298.2122. The interesting point is these numbers are resulted by the eight right angled Sierpinski triangles method. We can assume that these two elements should have such weights but for they are the heaviest and also radioactive elements, some of their masses reduced and transformed to energy. To guess the atomic weights of eighth period, with considering the calculated weights of the elements 117 and 118 by the equation 2, we decided to make slightly change on one coefficient on the equation 2. This change may be indicated as reduced mass released as radioactive energy. The resulted equation is

$$y = -8* 10\text{-}5x3 + 0.015x2 +1.7531x +1.4274 \quad ...(3)$$

The atomic numbers of eighth period and their atomic weights which are calculated by the equation 3 (Eq.3) are listed in table 6. However the elements of the eighth period of the periodic table have not yet discovered, so we can not determine their errors, but we can assumed their atomic weights and by this resulted equation and hope that their correlation with their atomic weight will be a kind of Sierpinski triangle fractals . The R- squared value for the numbers in table 6 is 1 which shows a complete correlation between the atomic numbers and their calculated atomic weights. If a discovered element shows a weight more or less than what is calculated in the table 6, we may relate it to the amount of masses which are transformed to energy.

CONCLUSION

Fractal geometries are found in nature and the whole world around us . One of this fractals is Sierpinski triangle. In this paper, with help of softwares such as MATLAB and Ecxel , we show that there is a correlation between the atomic numbers and atomic weights of the periodic table of elements and it is as right angle Sierpinski triagle fractals type. With making more triangle on the hypotenuse of the initial Sierpinski triangle, %errors will be reduced and calculated numbers will be closer to real ones. With the equations obtained with Sierpinski triangle algorithm we may guess the atomic weights of future discovered elements.

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