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An investigation of Hydraulic Conductivity Determination for Anisotropic Soils

MEHDI ZAMANI

Department of Technology and Engineering, Yasouj University,
Daneshjoo Avenue, Moallen Square, Yasouj, Iran.
*Corresponding author E-mail:

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ABSTRACT

One of the important parameters for determining the discharge through the subsurface system in groundwater engineering is hydraulic conductivity coefficient. This parameters the factor of oil skeleton and fluid properties. The horizontal and vertical components of saturated hydraulic conductivity coefficient can be obtained from the laboratory and field testing methods. Those are usually constant and variable head tests. In this research the three-dimensional ellipsoid of hydraulic conductivity coefficients is obtained. Using the hydraulic conductivity ellipsoid it is possible of obtain the permeability of soil in any direction. The above model has the effective application for calculation of one, two and three-dimensional fluid flow in the anisotropic porous media. The accuracy of thehydraulic conductivity obtained by the model can be checked by laboratory tests. The model is applied for the calculation of infiltration from the natural or man-made surface channels, also exfilitration from the subsurface water resources.

Key words: Hydraulic conductivity, anisotropy, permeability, variable head, constant head, infilitration.

INTRODUCTION

The natural unsolidified sediments and the compacted layers of soil in embankemnts and soil dams from rarely the horizontal stratifications and isotropic media. In those cases the soil masses are anisotropic and this should be considered for the calculations of fluid flow through them. In the horizontal stratification especially at the interfaces of layers the voids. The resistance of fluid flow in the horizontal direction is less for the above cases, therefore, the hydraulic conductivity coefficient in the horizontal direction is greater than the vertical direction. Also the situation of setting and the arrangement of soil particles due to the vertial compaction thin layers of coarse soil between the medium of thick layers of fine soils speeds the above process. As a general rule and for the most cases of soils the horizontal hydraulic conductivity coefficient is greater than the vertical one. The exception is for the fine Aeolian soils such as losses with silty particles that transported by wind. The random setting and packing of the silt particles on the top of each other somehow provides a system of voids that has more vertical pattern than the horizontal pattern.

The permeability is always important parameters in geotechnical engineering projects when the main concerns are the groundwater flood flow and contaminant transport through porous and fracture media.

2-one-dimensional fluid flow in two-dimensional anisotropic medium

In the following Cartesian coordinate system, Fig. (1) is s be the direction fluid flow and flux q_s be in the direction s, the components of velocity in directions x and y are,

$$q_x \quad q_s \, Cosa \;, \; q_y \quad q_s \; Sin$$

The orthogonal linear system of equations related to the rotation of coordinate system with the amount α respect to x axis is,

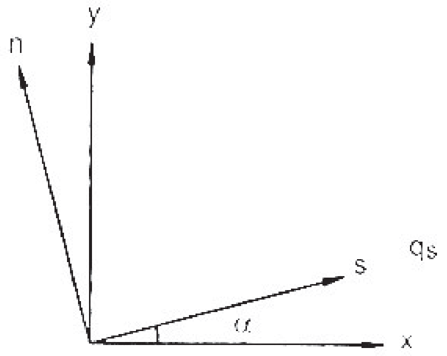

Fig. 1: Transformation of the coordinate system

...(2)

Where in Eq. (2)

y=s Sin a + n Cos a , x = s Cos a - n Sin a

...(3)

In Eq (3) K_x, K_y and K_s are the hydraulic conductivity coefficients in direction x, y and s, respectively and h is total head. With respecting to h=h (x,y), x =x (s,n) and y=y (s,n) for derivative of h respect to s it can be written as.

...(4)

If Eq. (4) is substituted in Darcy's equation for q_s

[illegible]

...(5)

Eq (5) can be written respect to qx and qy with applying Eq. (q) and (3) as

...(6)

Eq (6) can be simplified according to the angle of rotation α as

...(7)

Using Eq (7) it is possible to obtain K_s with regarding to K_x and K_y for any arbitrary angle α. The coordinates x and y can be written in the polar coordinate as x=r cos α and y=r sin α. Using the above expression for x and y in Eq. (7) the equation of the hydraulic conductivity ellipse is obtained from the following equation.

$$\frac{r^2}{K_s} \quad \frac{x^2}{K_x} \quad \frac{y^2}{K_y} \qquad \frac{x^2}{\sqrt{K_x}^{\;2}} \quad \frac{y^2}{\sqrt{K_y}^{\;2}} \quad \frac{r^2}{\sqrt{K_x}^{\;2}}$$

...(8)

Eq. (8) which is explained shortly by (Harr 1962, cedegreen 1977, Craig 2007), is the equation of ellipse with time major and semi minor diameters $\sqrt{K_x}$, and $\sqrt{K_y}$ respectively according to Fig. (2)

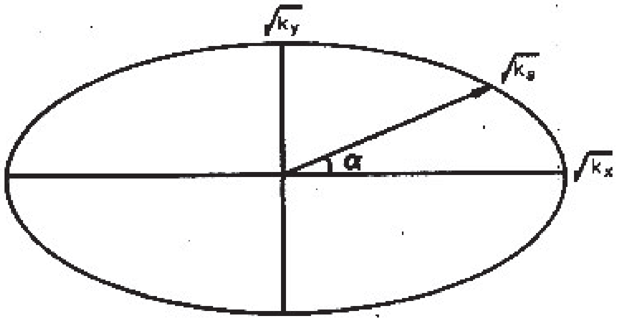

Fig. 2: The ellipse of the hydraulic conductivity

Regarding to the above figure the hydraulic conductivity coefficient in the direction a respect to x axis equals the square of the half diameter of ellipse which has the angle α with x axis or $K_s = r^2$. For the application of the above model one example is brought here.

Example 1

For the soil sample with anisotropy conditions as Fig. (3) obtain the horizontal hydraulic conductivity coefficient where is direction has an angle of α=50 Degrees with the direction of anisotropy Ks.

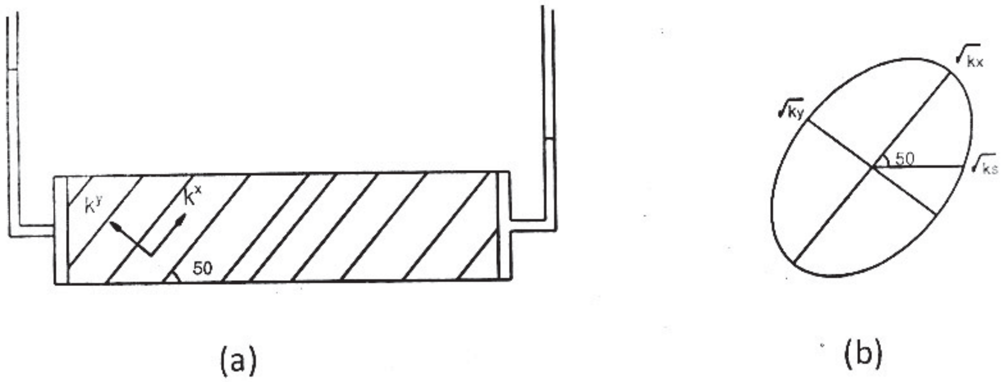

Fig. 3: Soil sample and its directional anisotropy, (b) the ellipse of hydraulic conductivity

$$\frac{1}{K_s} \; \frac{Cos^2 50}{K_x} \; \frac{Sin^2 50}{K_y} \; \frac{K_y\, Cos^2 50 \;\; K_x\, Sin^2 50}{K_x K_y} \quad ...(9)$$

Ks can be obtained from Eq. (9) as,

3. One dimensional fluid flow in three-dimensional anisotropic medium

The three-dimensional coordinate system is considered. Suppose fluid flow is one-dimensional and its direction is y" (s) which makes angle α from y axis and with the slope angle of β respect to the horizontal plane oxyz, fig (4).

The governing orthogonal system for the transformation (rotation) of oxyz coordinate system to o'x'y'z' coordinate system is,

$$\begin{matrix} x' & Cos & Sin & 0 & x \\ y' & Sin & Cos & 0 & y \\ z' & 0 & 0 & 1 & z \end{matrix} \quad ...(11)$$

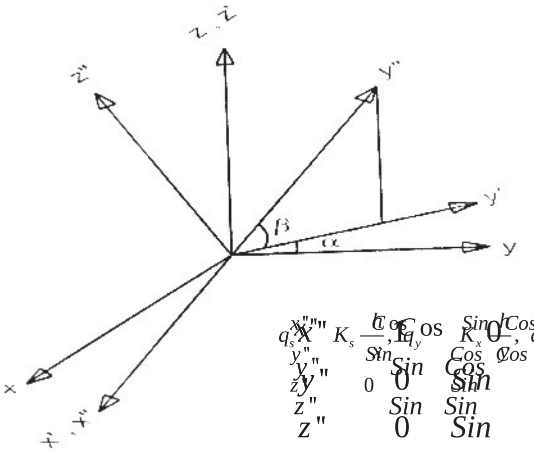

Fig. 4: Fluid flow and direction s (y")

The second transformation for the o'x'y'z' coordinate system to o"x"y"z" coordinate system requires the following orthogonal system

...(12)

Substitute Eq. (11) in Eq. (12) results,

...(13)

The components of Darcy's velocity vector in the directions x, y, z and s are obtained from the set of Eq. (14), (DeWiest 1965, Davis and De Wiest 1966, Todd and Mays 2005).

...(14)

In the system of equation (13) solve for vector x it results.

...(15)

In which,

...(16)

Applying Chain rule for the derivative of h respect to s,

...(17)

In Eq (17) is substituted in Darcy's law for the components it results,

...(18)

The derivatives of x, y and z respect to s with using Eq. (16) are,

...(19)

Eq. 20 can be obtained by substitution of the derivatives h respect to x, y and z,

...(20)

The components of flux or Darcy's velocity in directions x, y and z are,

...(21)

The Eq. (22) results after the substitution of Eq. (21) in Eq. (20) and simplification

...(22)

By the rearrangement and omitting the cancelled terms, Eq. (23) is obtained.

...(22)

With transferring the Cartesian coordinates terms, Eq (2) is obtained

$$\begin{aligned} x &\quad r\,Sin \quad Cos \\ qy &\quad r\,Cos \quad Cos \\ z &\quad rSin \end{aligned} \qquad ...(24)$$

Eq. (25) is obtained by substitution the equations (24) in Eq. (23). It is the equation of ellipsoid with the major axes in the direction of principal anisotropy.

[illegible] ...(25)

The major diameter of semi-axes in the above ellipsoid are $2\sqrt{K_x}, 2\sqrt{K_y}$, and $2\sqrt{K_2}$

...(26)

The above ellipsoid is shown in fig. (5)

The hydraulic conductivity coefficient in direction s from the above ellipsoid is.

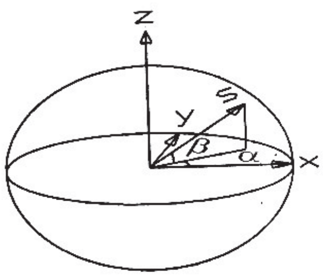

Fig. 5: The ellipsoid of hydraulic conductivity coefficients

$$K_x = r^2 = x^2 + y^2 + z^2 \qquad ...(27)$$

The situation when there is infiltrations from lakes with extended surface area, the flow is one-dimensional and in the vertical direction. For the case when the direction of conductivity coefficient in the direction of flow can be obtained from the above ellipoid. This is explained with some more details by the example2.

Example 2

Determine the amount of flux through a long horizontal column of soil sample as Fig. 6 where the cross-sectional dimensions are small respect to its length. The stratification of soil is along the octhaderal plane. Fig. (6a). The situation of the principal hydraulic conductivity coefficient in xyz coordinate system are shown in Fig. (6b). The anisotropic condition of the soil is considered transverse isotropic of $K_x = K_y > K_z$. The direction of flow in directions respect to the xyz coordinate system is shown in fig. (6b).

For the above condition α=60 degree and $\beta=\pi/2-\delta$, the hydraulic conductivity coefficient in the direction of flow is obtained eq. (28) where d=57.73 is the octhaderal angle.

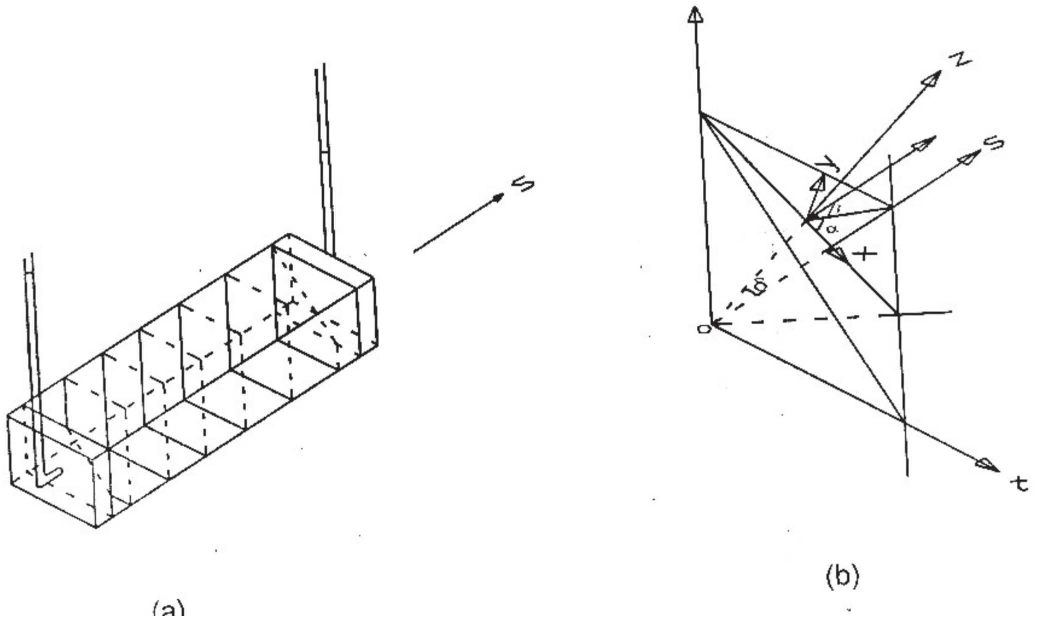

Fig. 5: (a) The horizontal column of soil (b) The directions of principal hydraulic conductivity

$$\frac{1}{K_x} \quad \frac{Sin^2 60\, Cos^2 36.26 \quad Cos^2 60\, Cos^2 35.26}{K_x} \quad \frac{Sin^2 35.26}{K_z} \qquad ...(28)$$

$$K_s \quad \frac{K_x\, K_z}{K_z\, Cos^2 35.26 \quad K_s\, Sin^2 35.26} \qquad ..(29)$$

Three dimensional fluid flow in three dimensional anistropic medium

This is the most general situation for flow of water through anistropic porous media. Here fluid is three-dimensional and has components x, y and z. In the most cases the velocity components q_x, q_y and q_z don't coincide with the principal directions of anistotrpy. If they do the relating system of equations for velocity components (Bear 1988) is,

$$\begin{matrix} q_x & K_s & 0 & 0 & h/\ x \\ q_y & 0 & K_y & 0 & h/\ y \\ q_z & 0 & 0 & K_z & h/\ z \end{matrix} \qquad ...(30)$$

In the above system K_x, K_y and K_z, are the principal hydraulic conductivity coefficients. If the velocity components for new coordinate system ox "y"z" are required. The governing transformation system for the rotation of oxyz coordinate system around angles α and β regarding to Fig. 4 is as follows.

...(31)

Usign Eq. (16) in the system of equation (31) Eq. (32) is obtained.

...(32)

The velocity components in the new coordinate system ox "y"z" are obtained from the projection of q_x, q_y and q_z in the new direction.

...(33)

Matrix A in the above system is orthogonal, therefore its inverse equals its transcope.

...(34)

If the vector {q} from Eq. (34) is substituted in Eq. (32).

$\{q''\} \quad [A'']\{q\} \qquad \{q''\} \quad [A]''[K][A]''\{\ h/\ \ x''\}$ ...(35)

For the vector {q") the Darcy's equation is,

...(36)

Where [K"] is the hydraulic conductivity coefficient matrix in the new coordinate system. Comparing Eqs. (35) and (36) [K"] equals.

...(37)

The matrix [A] in the system of equation (37) cna be changed into the following two-dimensional orthogonal matrix as,

Results and Recommendation

In this research the one, two and three-dimensional fluid flow for anisotropic porous media are investigated. The principal hydraulic conductivity coefficients when don't coined withthe Cartesian axis are reformulated. These coefficients are the input of calculation of fluxes across and beneath earth dams and concrete weighted dams, also for determination of exifilitration and infiltration from saturated lands, lakes and natural canals. The hydraulic conductivity coefficients tension is applied for numerical analysis specially, for finite element method in calculation of saturated fluid flow in subsurface soil system. If there are folding and bending in soil layers the hydraulic conductivity tensor varies with the distance therefore, the directions of principal hydraulic conductivity coefficients change with the location. This should be considered when it is supposed to analysis the fluid flow for the above conditions.

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