



## Three Dimensional Capacitor as Function of Voltage of Metal-Insulator-Electrolyte (MIE) and concentration of Electrolyte Plot for Ideal Case of MIE

KIUMARS GHOWSI\* and HOSEIN GHOWSI<sup>1</sup>

\*Department of Chemistry, Majlesi Branch, Islamic Azad University, Isfahan, Iran

<sup>2</sup>Department of Mathematics, Mobarekeh Branch, Islamic Azad University, Mbarekeh, Iran

\*Corresponding author: E-mail : k\_ghowsi@yahoo.com

(Received: October 01, 2011; Accepted: November 12, 2011)

### ABSTRACT

The total capacitance of the metal-insulator-electrolyte(MIE) has been studied theoretically. Mathematical model of the capacitance for MIE has been developed for ideal case. In the ideal case, a totally blocked interface has been assumed, such that the MIE behavior is governed only by electrostatic factors. Three dimensional plots for capacitance as a function of  $V_G$ , the potential between metal and electrolyte and the concentration of electrolyte is presented for the first time.

**Key words:** Double layer, Capacitance of metal-Insulator- Electrolyte.

### INTRODUCTION

Studies of the capacitance of the double layer was done for the first time by Graham<sup>1</sup>. Ofcourse these studies are focused on metal-electrolyte structure. In previous work<sup>2-6</sup> Metal-Insulator-Electrolyte structure were studied by us. It is common to study the interface of insulator-semiconductor in metal-insulator-semiconductor structure to use capacitance voltage plot<sup>7</sup>.

In present work we obtain the expression for capacitance of metal-insulator-electrolyte(MIE) structure, we plot the capacitance as a function of  $V_G$ , voltage between metal and electrolyte and,

$n^o$ , the concentration of electrolyte in three dimension. The ideal system of MIE is studied.

### Ideal MIE structure

The ideal MIE system is similar to what Siu<sup>8</sup> et al. have defined as the totally blocked interface of an insulator/electrolyte part of a semiconductor-insulator structure. In ideal MIE, there is a complete absence of interfacial reactions between the electrolyte and the oxide. Since the interfacial electrochemical processes are absent, the charge and potential distribution in this MIE system is dictated solely by electrostatic consideration. Furthermore, because the oxide exhibits no specificity to ionic species in the

electrolyte the response is determined by the total ionic strength , i.e. , the combined effects of all the ions in the electrolyte.

$V_o$  is the potential applied between metal and electrolyte. The charge per unit area and the potential in the electrolyte space charge region are related by the Poisson-Boltzman equation.

From Gauss's law and solution to this equation , we find that for an electrolyte the charge per unit area in the Gouy-Chapman space- charge region is given by

$$\sigma_d = (8\epsilon_e k T n^o)^{1/2} \sinh[q(V_G - \Psi_d) / 2kT] \dots(1)$$

To find the capacitance-voltage and concentration of electrolyte of the ideal MIE system since  $C_d$  the diffuse layer capacitance  $C_H$  the capacitance of the Helmholtz and  $C_o$  the capacitance of the oxide are in series , we have ,

$$C'_o = \frac{C_H \cdot C_o}{C_o + C_H} \text{ since } C_H \gg C_o, \text{ then } C'_o \approx C_o \dots(2)$$

... (3)

where  $C_o$  is the oxide layer capacitance ,  $C_H$  is the inner Helmholtz layer capacitance and is  $C_d$  the diffuse layer capacitance. We can find  $C_d$  by taking the derivative of equation (1) , as

$$C_d = -\frac{\partial \sigma_d}{\partial \Psi_d} = \left(\frac{2\epsilon_e n^o q^2}{kT}\right)^{1/2} \cosh\left[\frac{q(\Psi_d - V_G)}{2kT}\right] \dots(4)$$

where  $\epsilon_o$  is the dielectric constant of electrolyte,  $q$  is the electronic charge,  $k$  is the Boltzman constant  $T$  is the temperature and  $\Psi_d$  is the potential at the outer Helmholtz layer and  $V_G$  is the potential between metal and electrolyte. Substituting equation (4) into (3) , we get

$$C_T = \frac{C_o \left(\frac{2\epsilon_e n^o q^2}{kT}\right)^{1/2} \cosh\left[\frac{q(\Psi_d - V_G)}{2kT}\right]}{C_o + \left(\frac{2\epsilon_e n^o q^2}{kT}\right)^{1/2} \cosh\left[\frac{q(\Psi_d - V_G)}{2kT}\right]} \dots(5)$$

The difference  $(\Psi_d - V_G)$  is the zeta potential , so by inserting the equation obtained in previous work<sup>6</sup> for zeta potential equation (6) in to equation (5)

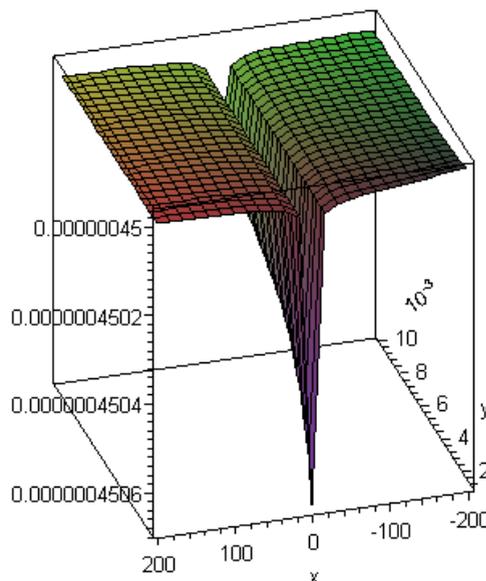
... (6)

we find  $C_T$  with respect to  $V_G$  .

$$C_T = \frac{-2\frac{kT}{q} C_o \left(\frac{2\epsilon_e n^o q^2}{kT}\right)^{1/2} \cosh \sinh^{-1}\left[\frac{V_G C_o}{(8\epsilon_e k T n^o)^{1/2}}\right]}{C_o - \frac{2kT}{q} \left(\frac{2\epsilon_e n^o q^2}{kT}\right)^{1/2} \cosh \sinh^{-1}\left[\frac{q(\Psi_d - V_G)}{2kT}\right]} \dots(7)$$

By using equation (8)

$$\frac{\partial C_T}{\partial V_G} = \frac{2kT C'_o}{C_o + C_o} \sinh^{-1} \left( \frac{V_G C_o}{(8\epsilon_e k T n^o)^{1/2}} \right) \dots(8)$$



**Fig. 1: Three dimensional plot of total capacitance of MIE, as a function of  $V_G$  and ,  $n^o$ , the electrolyte concentration**

and the definition of  $\cosh x = (e^x + e^{-x})/2$  by simple algebraic manipulation we find,

$$\cosh(\sinh^{-1} x) = x + \frac{1}{x + (x^2 + 1)^{1/2}} \quad \dots(9)$$

applying identity (9) to equation (7), it can be converted to

$$C_T = \frac{-2\alpha_1^2 \alpha_2^3 C_o^3 V_G^2 - 2\alpha_1^2 \alpha_2^2 C_o^2 V_G (\alpha_1 V_G C_o)^{1/2} + 1 - 2\alpha_1^2 \alpha_2 C_o}{C_o^3 \alpha_2 V_G + C_o (\alpha_1 V_G C_o)^2 + 1)^{1/2} - 2\alpha_1^2 \alpha_2^2 C_o^2 V_G^2 - 2\alpha_1^2 \alpha_2 C_o V_G (\alpha_1 V_G C_o)^{1/2} + 1)^{1/2} - 2\alpha_1^2 \alpha_2} \quad \dots(10)$$

where

$$\alpha_1 = \left( \frac{2\epsilon_e n^o q^2}{kT} \right)^{1/2}$$

$$\alpha_2 = \frac{1}{(8\epsilon_e kT n^o)^{1/2}}$$

$$\alpha_1 \alpha_2 = \frac{q}{2kT}$$

$C_T$  has been plotted three dimensionally as a function of  $V_G$  and  $n^o$  concentration of electrolyte in fig.1 by Maple 10 software for the first time. There a beautiful symmetrical groove shape is demonstrated. The range of  $V_G$  is between -200 volt to 200 volt and  $n^o$  is between 0.001 M to 0.01 M varies.

## CONCLUSION

There are two parameters which vary as a function of the  $V_G$  voltage between metal and electrolyte in metal-insulator-electrolyte (MIE) structure and  $n^o$  the concentration of electrolyte.

These two parameters are one zeta potential, the potential in the diffuse layer and the total capacitance of MIE structure. In reference 6 the zeta potential was plotted as a function of  $V_G$  and  $n^o$  three dimensionally for the first time for the ideal case where there is only electrostatic forces are present and surface ionization of the insulator is absent. In present work  $C_T$  the total capacitance of MIE has been plotted as a function of  $V_G$  and  $n^o$  in three dimension. A very nice symmetrical groove shape three dimensional surface results.

## REFERENCES

- O.C.Grahame, *Chem. Rev.*, **41**: 441 (1947).
- K.Ghowsi and R.J.Gale, in *Biosensor Technology*, R.P. Buck *et al.*, Eds. Marcel Dekker, New York, 55(1990).
- K.Ghowsi and R.J.Gale, *J. Chromatogr.*, 559: 95 (1991).
- K.Ghowsi and R.J.Gale, *ANLAB.* **23**, 17 (1991).
- K.Ghowsi and S.Naghshineh and MP Houline Russ. *J.Electrochem* **31**(12): 1259(1995).
- K.Ghowsi and H.Ghowsi, *Asian Journal of Chemistry* (In Press).
- S.M. Sze, physics of semiconductor Devices, New York, John Wiley and Sons(1981).
- W.M.Siu, R.S.C. Cobbold, *IEEE Trans. Electron Devices*, **26**: 1805 (1979).
- J.O'M. Bockris and A.K.N. Reddy, *Modern Electrochemistry Vols. 1 and 2*, New York, Plenum Press(1970).